The multi-periodicity of W Cygni

J. J. Howarth

The behaviour of the variable star W Cygni has been analysed from the past 89 years of BAA observations. Periods of approximately 131 and 234 days are evident, both being subject to apparently random shifts in phase and amplitude.

A report of the Variable Star Section *Director: J. E. Isles*

Introduction

The variable star W Cygni, RA 21h 36m 02s, Dec 45° 22' 29" (2000.0) (GC 30250, HD 205730, SAO 51079) has been on the programme of the BAA Variable Star Section since 1891. Although analysis has been undertaken on some of these observations, as described below, no attempt has been made to study with modern techniques the whole span of data which now extends to almost a century. This paper describes the computerisation of the BAA observations of W Cygni and the subsequent analysis of the light curve.

Background

The variability of W Cygni was reported in 1885 by J. E. Gore of Ballysodare, Co. Sligo, Ireland,[1] and confirmed by E. F. Sawyer of Cambridgeport, Mass., USA. The variable is assigned to the class of semiregulars SRb.[2] The spectral type has been variously reported to be from M4e to M6. Sky Catalogue 2000.0 gives M5IIIae.[3] In common with many other variables, the spectral class apparently becomes 'later' with decreasing light. Corresponding changes are seen in the IR.[4] Typical published values are: B —V = 1.6 and U —B = 1.3.

In the 1970s, W Cygni, along with many other late spectral type stars, was examined spectroscopically, in a search for molecular emission lines. Tests for water and hydroxyl ions proved negative,[5,6,7,8] though later work indicated a MgII flux and probable technetium.[10] Hagen et al. gave a mass loss rate of from 2 to 6 x 10^{-8} solar masses per year.[11] W Cygni evidently possesses a high gas to dust ratio in its envelope and does not show maser activity.

W Cygni is described by Eggen as a red giant of the Hyades Group and, as such, is among the oldest stars in the young galactic disk population - aged about 5 χ 10^{8} years.[12]

Prior analysis of variability

In 1919 Professor H. H. Turner and Miss Mary A. Blagg conducted an extensive study of the star based on BAA observations from 1907 to 1918, in which they deduced periods of 129.6 days of amplitude (half peak to peak) 0.36 mag, and 243 days of amplitude 0.26 mag.[13] A third possible variation of period 1944 days is also mentioned. The authors refer to 'halts' in the cyclic variations, which effectively represent sudden phase shifts. In 1920 the same authors wrote a second note taking into account observations by Sawyer (mentioned above) over the years 1885-1895. These extra observations confirmed the first two periods, but cast doubt on the third.[14] Further 'halts' were found.

Dinsmore Alter, in 1929, applied a correlation technique to both Turner and Blagg's data and later BAA data.[15] The established periods of Turner and Blagg[13,14] were increased to 132 and 249 days, and a further period of about 1100 days was noted. The dubious 1944 day period was not mentioned.

In 1969 the GCVS quoted from Matveev and Pereman a main cycle of 130.85 days, and a secondary cycle of 119.81 days.[16] In 1971 the GCVS Supplement quotes from Schubert a cycle of 126.26 days.[17]

In 1979 Klyus briefly reported the results of the statistical analyses of the light curves of six irregular and semi-regular stars.[18] For W Cygni he considered three series of results (with some overlap) for the years (i) 1907-1928, (ii) 1919-1940 and (iii) 1947-1960. From (i) he deduced two periods of 250.0 + 7.8 days (amplitude 0.20 mag) and 131.1 ± 2.2 days (amplitude 0.25 mag); from (ii) he deduced 235.3 + 6.9 days (amplitude 0.15 mag) and 131.1 + 2.2 days (amplitude 0.16 mag); and from (iii) he deduced 235.3 ± 13.9 days (amplitude 0.27 mag) and 129.0 ± 4.2 days (amplitude 0.19 mag). He attributed these changes either to the duration of the analysed series being too short or to a random character in the oscillations themselves. He considered systematic observational error to be a less likely cause.

In 1983 Klyus presented a more extensive paper based on the statistical analysis of time series, using a procedure which he had himself described.[19,20] He concluded that the light variation is most likely random, that is, although it exhibits cyclic variations, the amplitude, phase and frequency of the cycles change unpredictably with time. He did however discuss the alternative hypothesis that the light curve is actually composed of many interrelated harmonics (not necessarily multiples of each other) which give the appearance of randomness, whilst actually being deterministic.

More recently Szatmary, Mizser and Domeny performed a power spectrum analysis of the light curves of Y Lyncis and W Cygni, using Hungarian and Finnish observations made between 1973 and 1984, and concluded that three periodic variations were present simultaneously in both stars.[21] For W Cygni these were 1000 ± 50 days, 227 ± 5 days and 127 ± 2 days.

Observations

Prior to 1935 observations were available for most years from BAA *Memoirs*. For 1930-1932, however, no observations seem to have been published. Fortunately the observations of W. M. Lindley were available and these were regular enough to cover the gap. In the years before 1907 the data were somewhat sparse, with gaps occurring particularly in the first 4 months of the year when W Cygni is not well placed.

From 1935 onwards, unpublished sheets of observations were provided by the Variable Star Section, and these observations continued, albeit with a slight reduction in number during the War years, until 1987. For most of this period, W Cygni was extremely well covered, with typically 300 observations per year, the peak year being 1974 when nearly 800 were recorded. Some observations made by individuals in 1988 were also included. A grand total of 20 474 observations were digitised and processed.

From 1940 onwards a total of 13714 observations were available, made by 145 different observers. During this period the main contributors were as follows, each of whom made over 100 observations:

S. W. Albrighton	311
R. G. Andrews	764
J. R. Bazin	256
G. Broadbent	137
D. S. Brown	273
B. A. Carter	995
E. H. Collinson	334
D. P. Griffin	132
R. Griffin	204
F. M. Holborn	721
A. J. Hollis	317
D. Hufton	193
N. S. Kiernan	219
N. F. H. Knight	119
O. J. Knox	102
R. J. Livesey	873
R. S. Lomas	474
J. W. Macvey	304
A. Markham	143
E. Metson	108
I. A. Middlemist	396
P. A. Moore	288
B. R. M. Munden	114
M. J. Nicholls	134
N. Reid	131
T. A. Robinson	549
D. A. Rothery	111
T. G. Scott	254
J. D. Shanklin	271
I. H. Stanley	145
G. E. B. Stephenson	270
D. M. Swain	104
A. Tanti	216
M. D. Taylor	374
J. Toone	258

Thanks are especially due to these individuals, and to W. M. Lindley, who contributed a long series of estimates, mainly prior to 1940.

In a preliminary study, the date and magnitude of the 3680 observations in the *Memoirs* from 1900 to 1924 were computerised; then they were divided into 5-year series starting in 1900, 1905, 1910, 1915 and 1920. Fourier analysis showed two periods, one of about 130 days and one of about 240 days to be present in all groups. Interestingly in the series starting in 1905, the 240 day period had about twice the amplitude of the 130 day period; yet in the next 5-year series the opposite was true; and from 1915 to 1920 the amplitudes were about equal. These marked variations encouraged a more extensive investigation into the whole data set available.

Method

The tabulated observations were distributed amongst 12 individuals who had volunteered to be 'typists'. Using a variety of personal computers the observer, date and deduced magnitude were recorded. For much of the data the estimate, instrument, class, sky and time were also available. It was felt, however, that the extra effort involved in recording these would not, at this stage, be justified. Although the task of digitisation was an onerous one, it proved to be quite practicable. It was found that approximately 300 observations per hour could be handled by a practised 'typist' (12 seconds per observation), which amounted to about 6 hours of work for each person. Checking could be done quite quickly and effectively by eye, when most serious errors stood out from the rest.

The data from each individual were collected together on an IBM PC where the analysis was to be done. As a further check the computer was programmed to scan the data for invalid dates, and magnitudes that lay outside a certain range.

The analysis was begun by pooling the data into 20-day means, without attempting to weight or adjust the data according to the observer. It was felt that there was a sufficient density of observations for differences between individuals to effectively cancel out. Since the observer is recorded, this can be done, if considered necessary, at a later date. The 1469 20-day means had an average value of 6.18 magnitude, a standard deviation of 0.37 magnitude, and ranged from 5.27 to 7.34 magnitude.

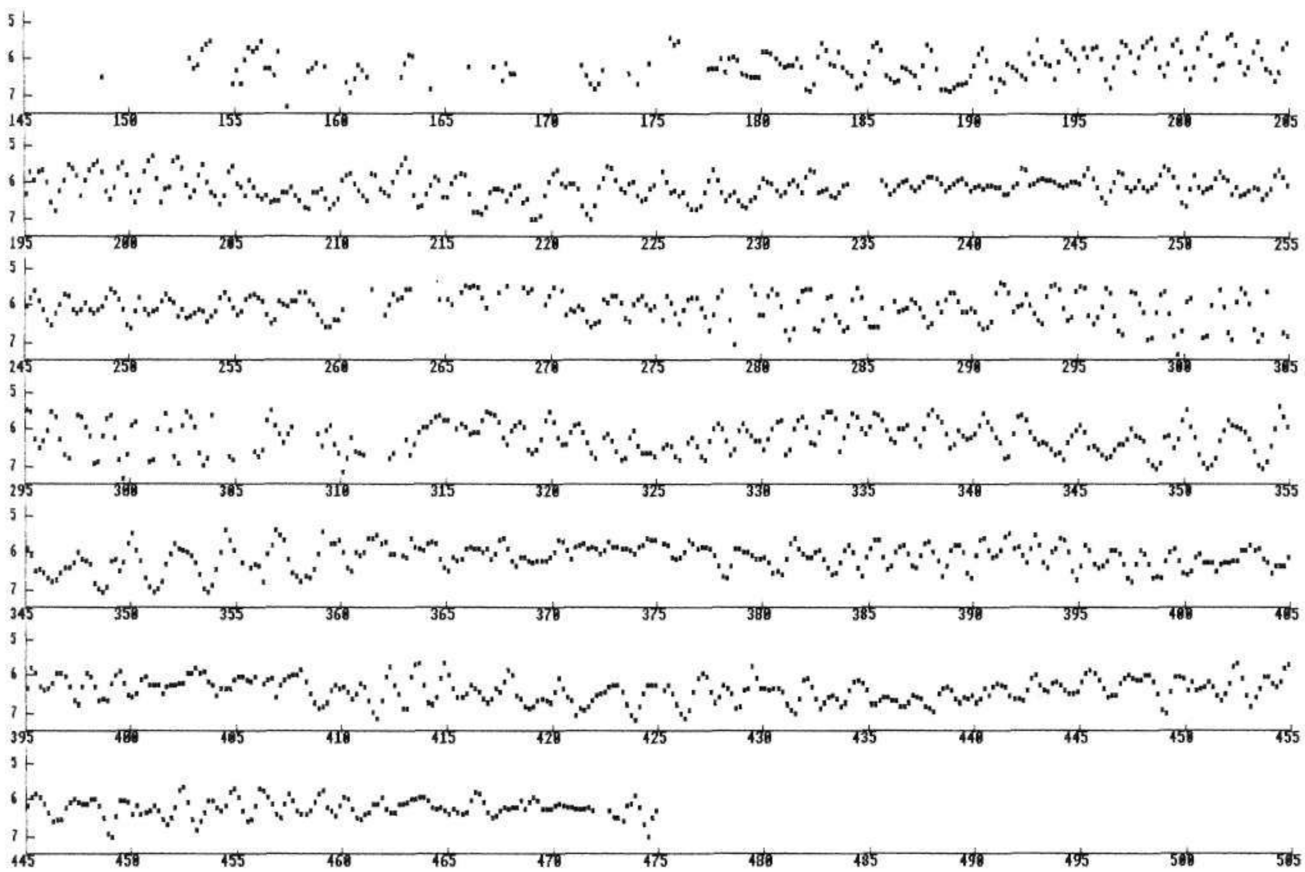


Figure 1. Graph of magnitude (20-day means) versus time. The folded time axis is annotated in units of 100 days, with the 2 most significant digits of the Julian date (24) omitted. For example, 165 corresponds to JD 2416500. Each strip spans about 16 years of data.

Next Fourier analysis was performed on the 20-day means. This is a technique for determining which periodic oscillations are present in a set of data in a time sequence. If the times of the η observations are t_1 $t_2,\ldots,$ t_n, and the corresponding magnitudes are m_1 ..., m_n, first we calculate

$$\mu = \frac{1}{n} \Sigma^n_{i=1} m_i .$$

Then we calculate, for a test frequency f_k,

$$C_k = \Sigma^n_{i=1} (m_i - \mu)\cos(2\pi f_k t_i)$$
$$S_k = \Sigma^n_{i=1} (m_i - \mu)\sin(2\pi f_k t_i)$$
$$\text{and } R_k = \frac{4}{n^2} (C_k^2 + S_k^2).$$

R_k is a measure of the amplitude of the frequency f_k in the light curve. As n becomes large, and assuming the t_i are randomly placed relative to the phase of the oscillation, then the amplitude *of* f_k is estimated as

$$a_k = VR_k \quad \bullet$$

A range *of* f_k was tested, from 0.00001 to 0.02 cycles per day (cpd) in intervals of 0.00001 cpd. Thus 2000 frequencies were tested in all. The chosen resolution of 0.00001 cpd is fine enough to ensure that, with an 89 year duration for the data, no frequency could fall undetected between two successive f_k. The maximum frequency tested corresponds to a 50-day period.

It was of interest to know whether any detected periods changed, with time, in amplitude or phase. There was also the possibility of change of frequency, but this would manifest itself simply as a variation in the rate of change of phase with time. In order to look for changes a 'moving window' examination was done. After sorting the 20-day means into chronological order, the data were copied into overlapping bins each of length t_w days, each bin starting t days in advance of the previous, so each 20-day mean could appear in many bins. Each bin could be thought of as a time 'window' on the star's behaviour. When this was complete, bins containing less than ten 20-day means were disregarded, since it was not possible sensibly to determine phase or amplitude, of two periods, from such small numbers of data.

For each bin, a curve fit would be carried out of the form

$$m_i^* = a_1 \sin(2\pi F_1 t_i + \varphi_1) + a_2 \sin(2\pi F_2 t_i + \varphi_2) + b_i^*$$

where each predicted magnitude m^* is as close as possible to the observed 20-day mean, m_r

The quantities a_v a_2, φ,, φ_2 would be calculated so as to minimise the summed least squares error

$$\Sigma^n_{i=1} (m_i^* - m_i)^2$$

over the bin. The quantities F_l and F_2 would be the two principal frequencies of variation (in cpd) that were detected. The estimated amplitudes for the window of

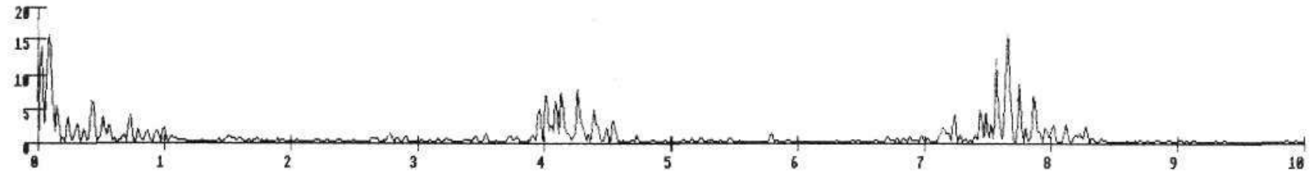


Figure 2. Graph of squared amplitude versus frequency: vertical axis is in units of 0.001 magnitude squared; horizontal axis is in units of 0.001 cycles per day. Data for frequencies greater than 0.01 cycles per day are not plotted, as no significant peaks were found in this region.

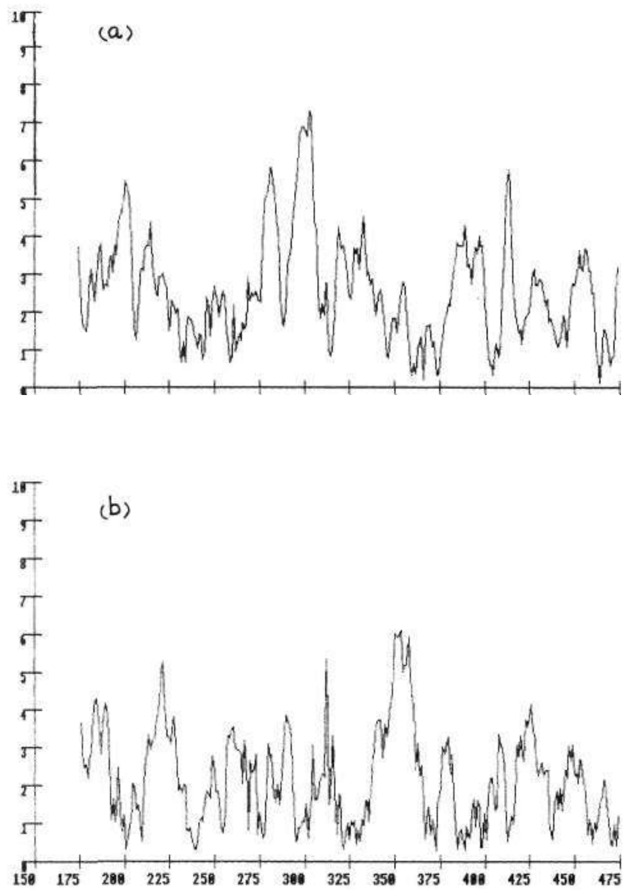


Figure 3. Graphs of amplitude versus time for (a) the 130.52 day period, (b) the 234.35 day period. Amplitude is in units of 0.1 magnitude, and the time axis is annotated as in Figure 1.

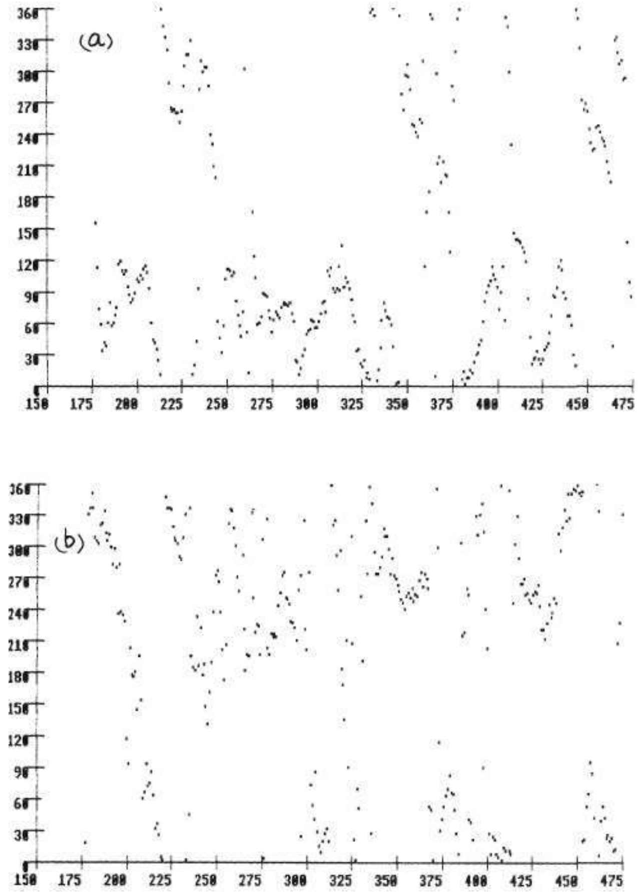


Figure 4. Graphs of phase versus time for (a) the 130.52 day period, (b) the 234.35 day period. Phase is in degrees and the time axis is annotated as in Figure 1.

the variations at these frequencies are a_1 and a_2, and the estimated phases are φ_1, and φ_2. The quantity b_i^* is an estimate of the residual magnitude at the time of the ith 20-day mean, that would remain if the two periodicities were subtracted out. It is itself estimated by applying a low-pass filter of time constant t_f to the series of 20-day means. The time constant is chosen to be long enough to filter out the cyclic variations F_1 and F_2, whilst still allowing b_i^* to respond to any secular changes, or to any long term variations.

With this technique it was intended to monitor the phase and amplitude behaviour of each of the two periods separately.

Results

The 20-day mean magnitudes are plotted in Fig. 1. The horizontal axis is time expressed as hundreds of days since JD 2400000. (Thus 165 corresponds to JD 2416500.) Each graph has a range of 2500 days, or just under 7 years. By eye we can identify regions where a period of about 130 days predominates (e.g. 295-305); regions where a period of about 240 days predominates (e.g. 182-187, 342-360); regions where 130 is present but weaker (e.g. 236-240); and regions where both are clearly present (e.g. 285-393). In addition to the periodic variations the star exhibits occasional longer faint periods (e.g. 420-425) and bright periods (e.g. 262-270), though elsewhere its average is close to the overall mean of 6.18 mag. There are also times when neither period is conspicuous (e.g. 373, 471).

Fig. 2 shows the results of Fourier analysis performed on all the 20-day means: that is, the graph of R versus f. The f axis shows frequency in units of 0.001 cpd, and the R axis is in units of 0.001 magnitude squared. It can be seen that there are two main clusters of peaks, in the regions of 0.004 cpd and 0.0075 cpd.

These clusters are indicative of a central frequency which varies in amplitude, phase or both, either randomly or deterministically. Both of these possibilities are discussed by Klyus.[19] The central peaks of the clusters are situated at 0.007662 cpd (130.52 days) and 0.004267 cpd (234.35 days). The amplitudes are 0.126 and 0.090 mag respectively. It should be kept in mind that these represent average half peak to peak deviations, in effect, the mean excursion taken over the entire observation time span. These amplitudes are less than those given by Turner and Blagg[13] and Klyus[18] because, in the extensive period studied here, there has been much variation in phase and period, leading to considerable cancellation. To put it another way, instead of appearing as a single large peak, the power is spread over the cluster of peaks that we can now resolve.

Multiples of both periods (harmonics) were also tested for, but proved to be insignificant. The first, second and third harmonics of the 130.52 day period had amplitudes 0.009, 0.004 and 0.005 mag respectively and of the 234.35 day period had amplitudes 0.008, 0.006 and 0.001 mag. Thus, both the central peaks appear to be essentially sinusoid in shape. This probably applies to all the side peaks also, since the mechanism by which these are generated is likely to be the same as the central periodicity.

As a further experiment, the exact period and amplitude of each central peak was determined after subtracting out ('whitening') the other central peak from the data. As this did not affect either period significantly, it was concluded that the peak frequencies, and hence the clusters, were independent.

Additionally there is some evidence of periodicities of more than 1000 days. Although the peaks are substantial, they must be treated with scepticism when the period approaches the time span of the data. Nevertheless, the peaks at about 0.00073 and 0.00039 cpd (1370 and 2540 days) are worthy of note.

For the 'moving window' examination the central frequencies of $F_1 = 0.007662$ and $F_2 = 0.004267$ cpd were used. The other parameters chosen were $t_w = 400$, $t_s = 100$ and $t_f = 400$ days. Figs. 3a-b show the amplitudes versus time for the two periodicities: the axes show estimated amplitude in units of 0.1 mag against Julian date, scaled as in Fig. 1.

Fig. 3a shows an amplitude surge approximately every 10 000 days (about 27 years), at about 200, 300 and 400 on the time axis; Fig. 3b shows a tentative modulation envelope having a period of roughly 3000 days (about 8 years), with peaks at around 180, 220, 260, 290, 305, 350, 380, 410, 425 and 450. Fig. 4 shows the phase of the data relative to (a) the 130.52 and (b) the 234.35 day period, with the vertical axis in degrees. The top and bottom boundaries of both these graphs represent the same phase and can therefore be joined: the graphs might thus be regarded as cylinders. It is worth describing the phase behaviour that could give rise to certain features of these graphs: a steady period of 130.52 days (in Fig. 4a) would give a straight horizontal line; a steady period of less (more) than this would give a line that sloped upwards (downwards); a decreasing (increasing) period would cause the graph to curve upwards (downwards); an abrupt change in phase would cause a step jump to occur in the graph, while an abrupt change of period would cause the line to change gradient. These effects will be smoothed out slightly due to the window size. All these phenomena are actually found in the O — C curves of, say, eclipsing binary stars, though the mechanism that causes them is, of course, quite different from that operating in W Cygni.

Data before 175 has not been included in Figs. 3 or 4, owing to sparsity of observations before 1907, and an unavoidable delay before the low-pass filter settled.

Fig. 4a shows many striking variations in frequency. In particular there is a tendency, every so often, for the period to switch from below the average value to above it, giving an upward pointing cusp. Moreover these cusps often appear in pairs, for example, at 190 and 205, 320 and 335, and 395 and 430. There is a faint suggestion of a cyclic change in frequency of, perhaps, about 10000 days. The gain in period centred on 215 begins with a sudden fall at about JD 2420900, which is the long 'halt' between Jan 13 and May 22, 1916, mentioned by Turner and Blagg.[13]

Fig. 4b appears at first glance to be totally chaotic, but it too shows cusps, which correspond to changes in period about the mean (234.35 days), just as in Fig. 4a. At 355 and at 425 the phase plot appears to dip and then climb again, indicating a sudden reduction in period. A similar event occurs near 270, perhaps suggesting a periodicity in phase behaviour of approximately 7000 days, though there is no evidence of this before 270.

Conclusions

The analysis confirmed the presence of two independent periodic variations of average period 130.52 and 234.35 days, and of average amplitudes (half peak to peak) 0.126 and 0.090 mag respectively. Both appear to be essentially sinusoid in shape when plotted as stellar magnitudes. It should be stressed, however, that both the period and amplitude of these cycles are subject to change - so the values given here are strictly only estimates for the duration of the data. Other periods given in the literature are not confirmed, but 2 further periods of about 1400 and 2500 days were noted. It remains to be seen whether these are permanent features.

The variations in amplitude and frequency, and the shifts in phase from time to time, appear to be largely random. However this may be because the true behaviour is extremely complex, giving the appearance of randomness. The phase changes in the 130.52 and 234.35 periods show some evidence of cyclic recurrence.

Acknowledgements

Thanks are due to all those dedicated observers who have contributed their estimates over the years to the BAA VSS. The digitisation of the data was undertaken by members and friends of CMHAS, and it is hoped that this investigation will encourage other amateur astronomical societies to undertake similar work. Particular thanks are due to the officers of the VSS and to Dr David Stickland (RAL) for their help and encouragement with this project.

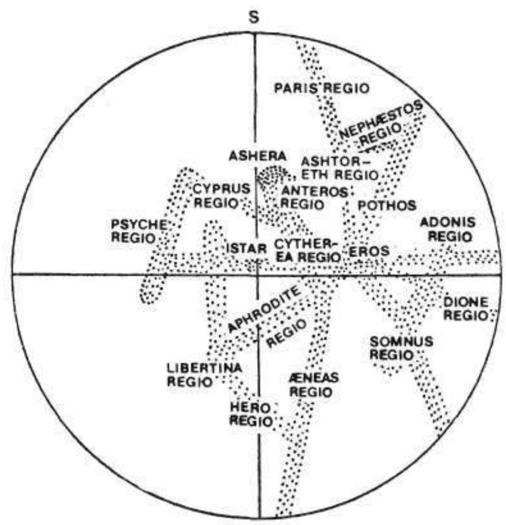
Address: Crayford Manor House Astronomical Society, Crayford Manor House, Mayplace Road East, Crayford, Kent. DA1 4HB

References

1 Gore, J. E., *Astron. Nachr.,* 122, 319, 1885.
2 Kholopov, **P.** N. (Ed.), *General Catalogue of Variable Stars, Fourth Edition, II,* Moscow, 1985.
3 *Sky Catalogue 2000.0, Vol. 2,* 1985.
4 Movsesyan, T. A., *Naucnye Informacii,* 56, 113, 1984.
5 Forbes, F. F. et al., *Astron. J.,* 75, 158, 1970.
6 Dickinson, D. F. et al., *Astrophys. Journal,* 180, 831, 1973.
7 Dickinson, D. F., *Astrophys. Journal Suppl. ser.* 30, 259, 1976.
8 Dickinson, D. F., and St Clair Dinger, A., *Astrophys J.,* 254, 136, 1982.
9 Stencel, R. E. et al., *Astrophys. J.,* 308, 859, 1986.
10 Little, S. J., *Astron. J.,* 194, 981, 1987.
11 Hagen, W. et al., *Astrophys. J.,* 274, 286, 1983.
12 Eggen, O. J., *Astrophys. J.,* 158, 225, 1969.
13 Turner, H. H. and Blagg, M. A., *Moη. Not. R. Astron. Soc,* 80, 41, 1919.
14 Turner, H. H. and Blagg, M. A., *Moη. Not. R. Astron. Soc.,* 81, 144, 1920.
15 Alter, D., *Astron. J.,* 40, 3, 1929.
16 Matveev, I. V., *Peremen. Zvezdy, 1* (6), 330, 1951.
17 Schubert, H., *Mitt. BAV,* 19 (1), 7, 1970.
18 Klyus, I. A., *Astron. Tsirk.,* 1070, 1, 1979.
19 Klyus, I. A., *Astron. Zh.,* 60, 320, 1983.
20 Klyus, I. A., *Peremen. Zvezdy,* 21, 115, 1978.
21 Szatmary, K. et al., *BAA VSS 'Light Curve',* 62, 2, 1985.

Accepted in revised form 1990 February 28.

Percival Lowell and the 'Canals' of Venus



Lowell's 1897 map of Venus.

What did Percival Lowell see on Venus? Was he misled by a subtle illusion? Or do his observations indicate the existence of an unrecognized dynamical feature of the planet's atmosphere?

Lowell first announced his findings with the 24-inch Flagstaff refractor in October 1896. 'The markings', he reported to the Boston Scientific Society, 'proved surprisingly distinct; in the matter of contrast as accentuated, in good seeing, as the markings on the Moon and owing to their character much easier to draw'. It was a bold, uncompromizing statement. But it merely signalled an even stranger disclosure. For in reference to the markings he further remarked: 'A large number of them, but by no means all, radiate like spokes from a certain centre'. According to Lowell then, Venus was characterized by a set of markings as distinctive of its physical appearance as the canals were of Mars; long, fingerlike streaks 'which started from the planet's periphery and ran inwards to a point not very distant from the centre ... well-defined and broad at the edge, dwindling and growing fainter as they proceeded, requiring the best of definition for their following to their central hub'. The arrangement was unique in the observational history of Venus.

Not unexpectedly Lowell found himself isolated and under attack. 'Mr Lowell's observations', opined John Ritchie Jr in the *Boston Evening Transcript* of November 28 1896, 'will not be accepted by astronomers as final'. While Camille Flammarion noted they are: '.. . entirely at variance with all that has gone before'. In 1903 Lowell too had doubts. But by 1907 these had been dispelled, and by 1914 he claimed: '... the radial markings ... exist all round the planet's disk'. Observations directly confirmatory of Lowell were made by Maxwell (1916), Wilson (1916 and 1917), Seagrave (1919), and Barker (1934). Camus (1932) and Dollfus (1948-1953) also depicted a radial pattern, but one more consistent with the diffuse, cloudlike markings normally observed. Even so the spoke system is still largely unconfirmed, and thought to be illusory.

Now however the subject is again centre stage. A new study by Dr J. J. Goldstein, National Air and Space Museum, Washington DC, though not intended to argue the reality of the Lowellian markings, does nevertheless suggest: '... that they are curiously consistent with observed dynamical symmetry'. Dr Goldstein concludes that their dismissal was premature, and that a search for them, using verifiable means, is completely justified. Dr Goldstein's study was published in May 1990 as Appendix 4 to NASA Contractor Report 4290, *Absolute Wind Measurements in the Lower Thermosphere of Venus Using Infrared Heterodyne Spectroscopy.* A revised paper has been submitted to *Icarus.*

Richard Baum